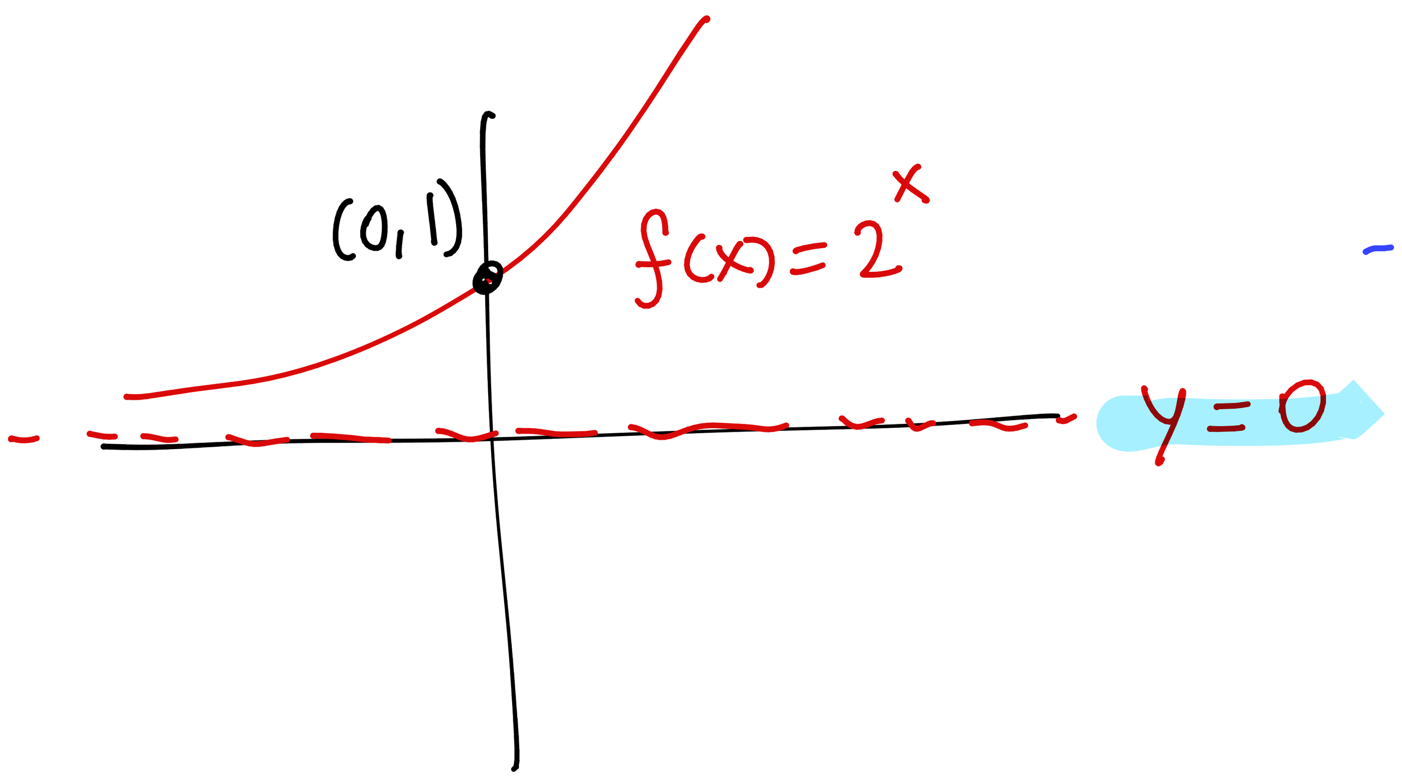
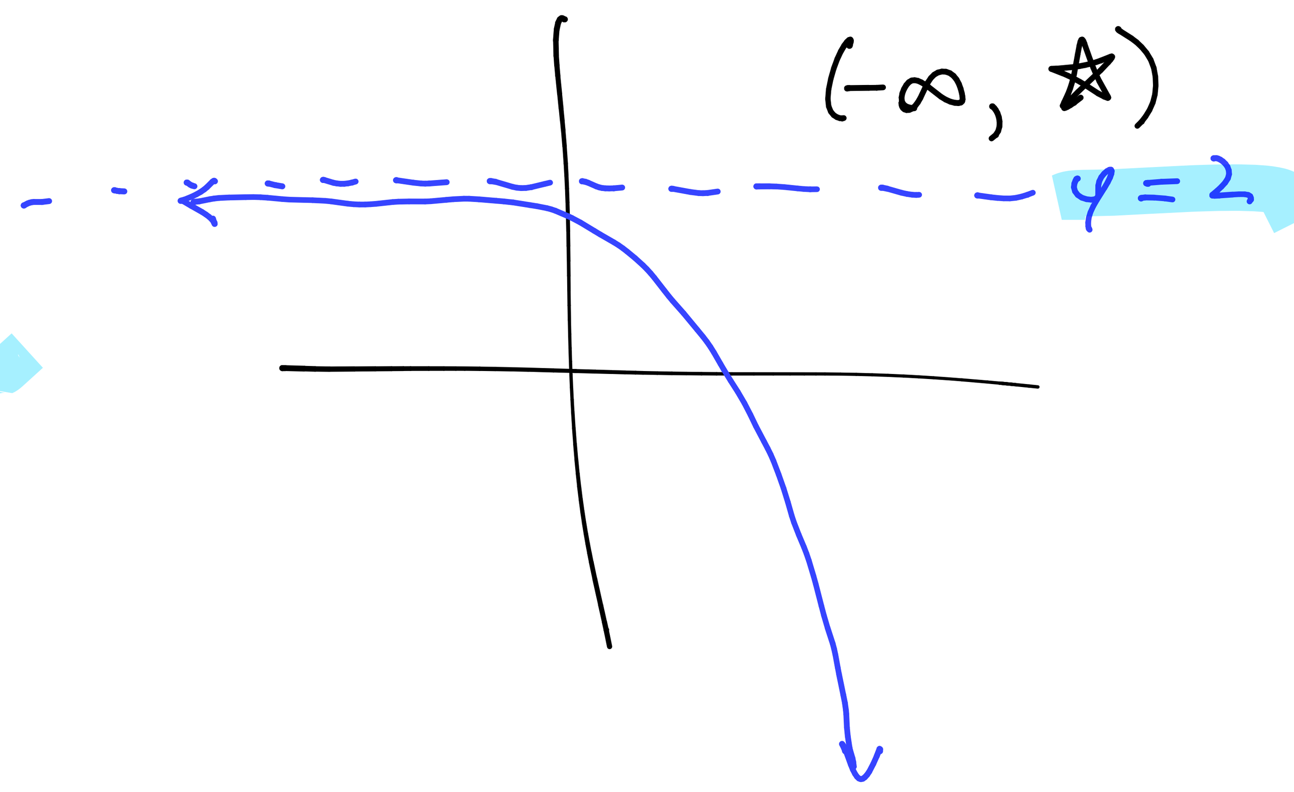
Class 18 - Logarithmic Functions

Review

Exponential functions: $(-\infty, \infty)$ Domain   Range $(\star, \infty)$ $(-\infty, \star)$





$(0, \infty)$ Range

$f(x) = -3^x + 2$. Range: $(-\infty, 2)$

	Example	Domain	Range
Exponential	$f(x) = b^x$, $b>0$, $b \neq 1$	$(-\infty, \infty)$	$(0, \infty)$
Inverse of exp. Logarithmic	$f^{-1}(x) = \log_b (x)$	$(0, \infty)$	$(-\infty, \infty)$

- Negative exponents & Fractional exponents:

$$\frac{1}{100,000} = 10^{-5}$$

$$(10^5)^{-1} = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = (100,000)^{-1} \Rightarrow 10^{-5} = \frac{1}{100,000}$$

$$81^{\frac{1}{4}} = \sqrt[4]{81^1} = \sqrt[\cancel{4}]{3^{\cancel{4}}} = 3 \qquad 81^{\frac{3}{4}} = \sqrt[4]{81^3} = \sqrt[\cancel{4}]{(3^{\cancel{4}})^3} = 3^3 = 27.$$

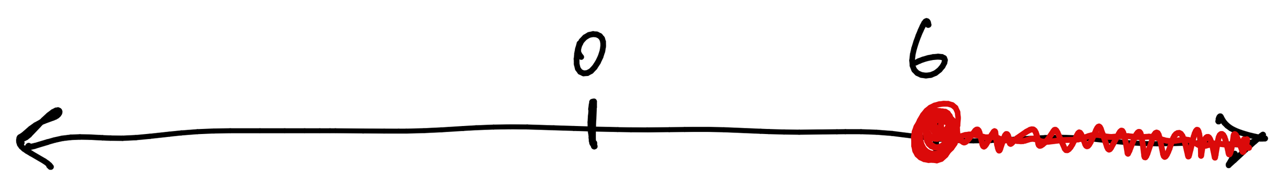
→ Linear inequalities:

What is the domain of $f(x)=\sqrt{x-6}$?

↳ Same as: "When we can take the square root of $x-6$?"

$\sqrt{\star}$ only exists when $\star \geq 0$.

Domain of $\sqrt{x-6}$ is all #'s such that $x-6 \geq 0 \iff \boxed{x \geq 6}$



or $\{x \mid x \geq 6\}$ or $[6, \infty)$

$g(x)=\log_b(x+3)$. Domain: $x+3>0 \iff \boxed{x>-3}$ or $(-3,\infty)$.

Solve $10-2x>0$.

① $10-2x>0$

$-10 \qquad\qquad -10$

$\frac{-2x}{-2} > \frac{-10}{-2}$

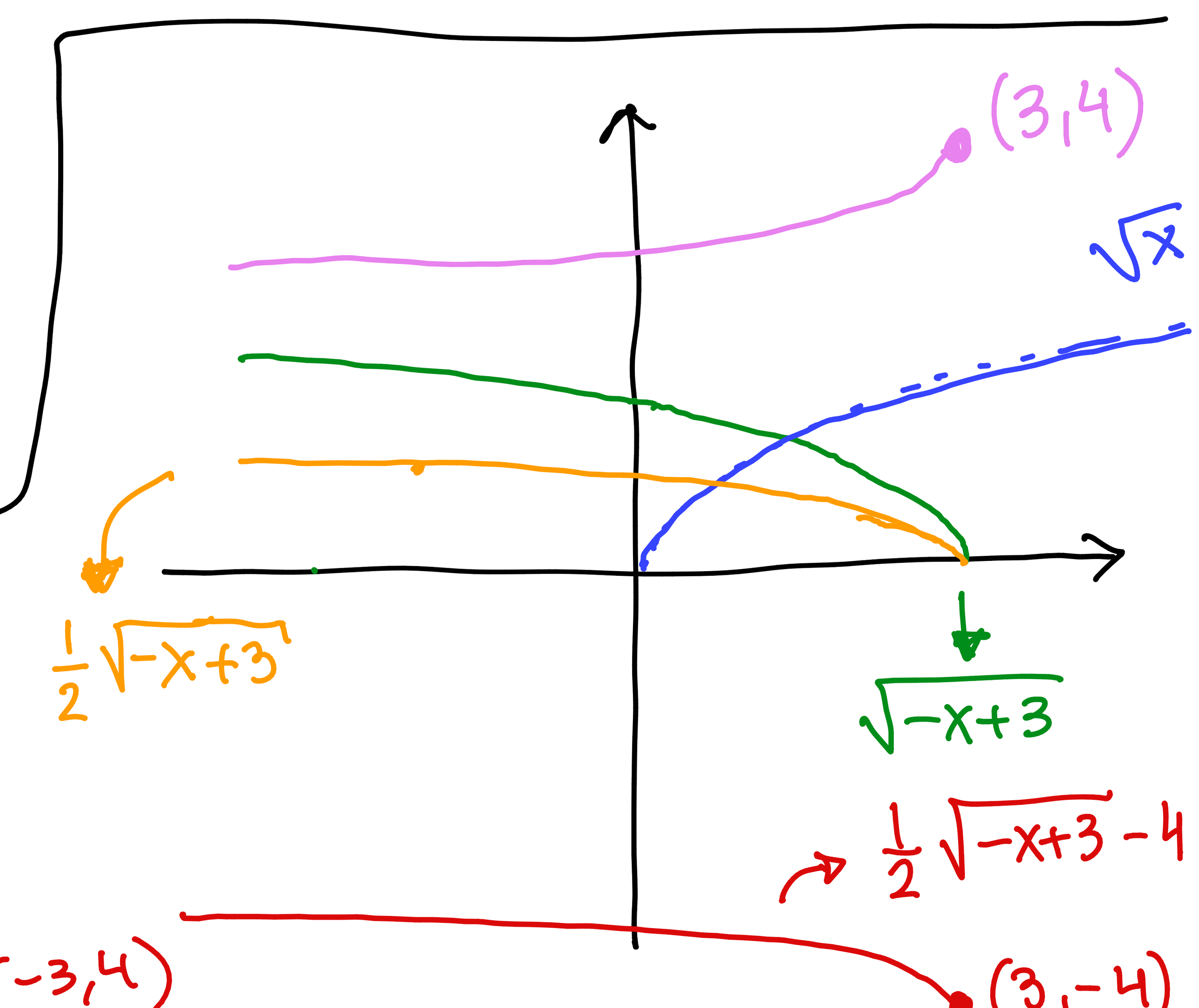
$\boxed{x<5}$

② $10-\cancel{2x}>0$

$+\cancel{2x} \qquad +2x$

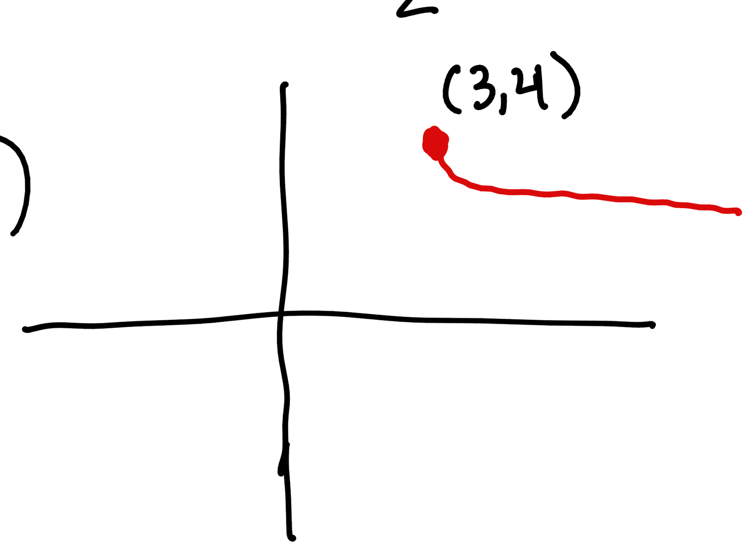
$\frac{10}{2} > \frac{2x}{2}$

$\boxed{5>x}$



→ Transformations of Graphs:

$f(x)=-\frac{1}{2}\sqrt{3-x}+4 = -\left(\frac{1}{2}\sqrt{-x+3}-4\right)$

a)


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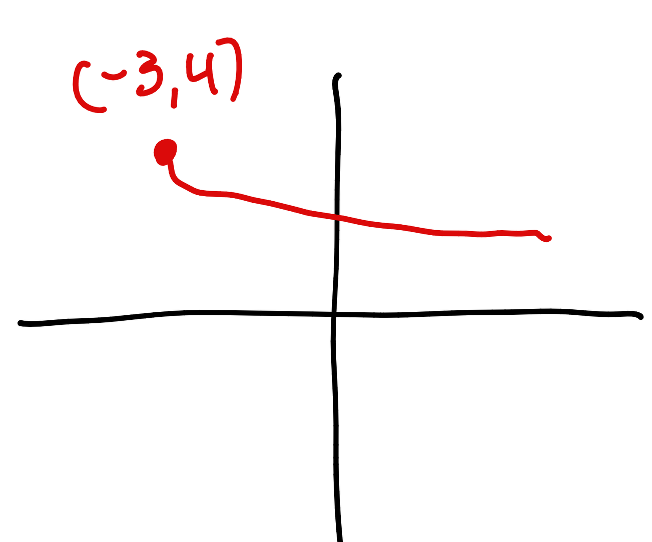

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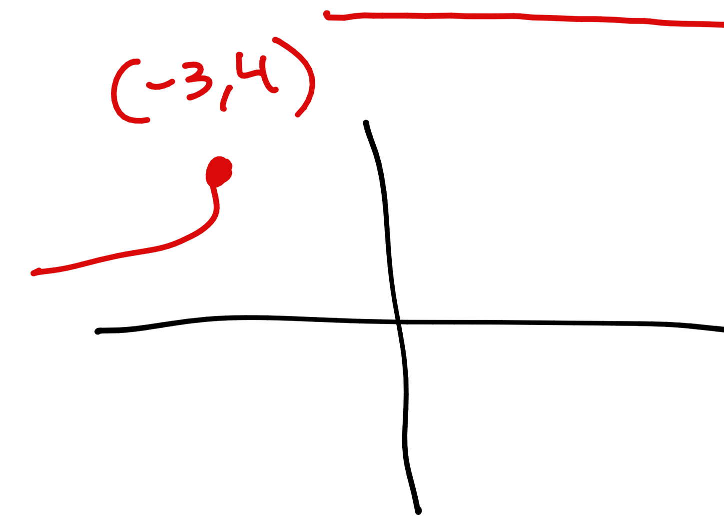

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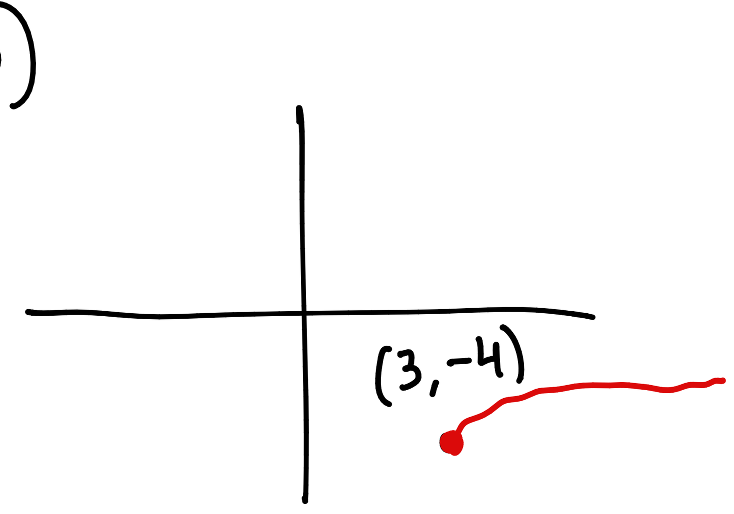

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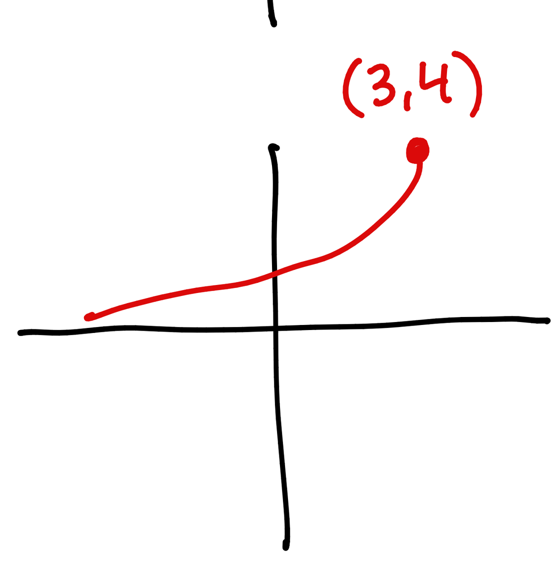
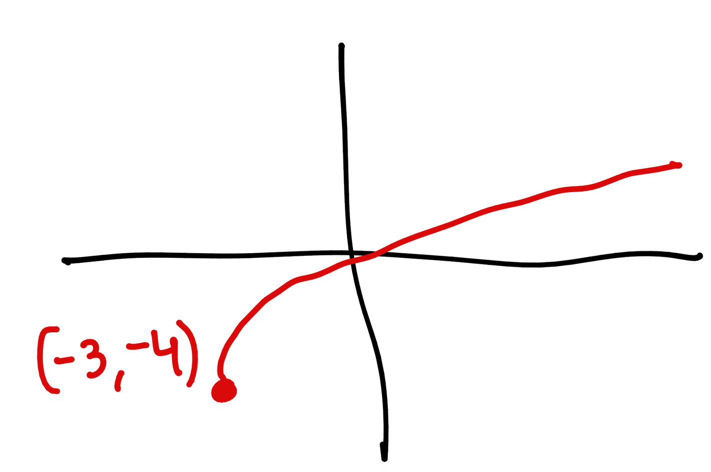


f)


① Basic fun.

② Inside fun.

③ stretch/comp.

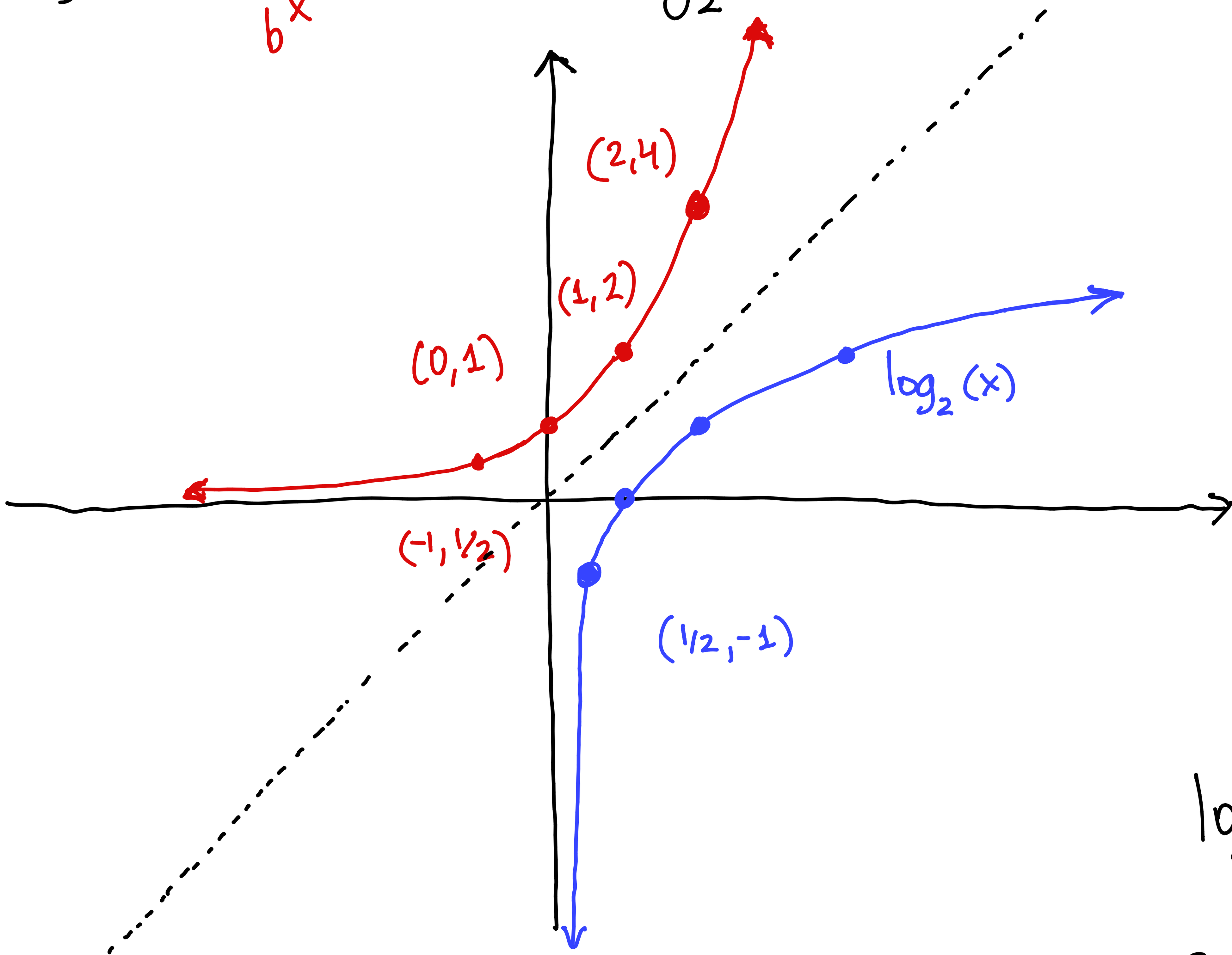
④ Outside fun.

⑤ (−) factored

Logarithmic Functions

$f(x) = 2^x \longrightarrow \log_2(x)$

b^x



$$\boxed{y = \log_b x \iff b^y = x}$$

Examples:

$3 = \log_2(8) \iff 2^3 = 8$

$\log_2 8$ means the exponent y s.t. $2^y = 8$

$$\boxed{y = \log_b x \iff b^y = x}$$

Ex 1: $49^{\frac{1}{2}} = 7 \iff \log_{49} 7 = \frac{1}{2}$

Ex 2: $7^4 = 2401 \iff 4 = \log_7 2401$

Ex 3: $\log_8 4096 = 4 \iff 8^4 = 4096$

Ex 4: $\log_5 K = L \iff 5^L = K$

General Properties of Log's

Remember: $b \neq 1$ & $b > 0$

1) $b^1 = b \iff \log_b b = 1$. $\log_5 5 = 1$, $\log_{10} 10 = 1$, $\log_{2025} 2025 = 1$, $\log_e e = 1$.

2) $b^0 = 1 \iff \log_b 1 = 0$. $\log_5 1 = 0$, $\log_{10} 1 = 0$, $\log_{2025} 1 = 0$, $\log_e 1 = 0$

Cancellation Properties of Log's

Recall: $\sqrt{x^2} = (\sqrt{x})^2 = x$, $x \geq 0$. $f(g(x)) = g(f(x)) = x$

Let $f(x) = b^x$, $g(x) = \log_b x$. Then:

1) $b^{\log_b x} = x \rightarrow f(g(x)) = b^{g(x)}$. $\cancel{2}^{\cancel{\log_2} 50} = 50$, $\cancel{2}^{\cancel{\log_2} 1{,}000} = 1{,}000$

2) $\log_b b^x = x \rightarrow g(f(x)) = \log_b(f(x))$. $\cancel{\log_5 5}^{20} = 20$, $\cancel{\log_{80} 80}^{2} = 2$

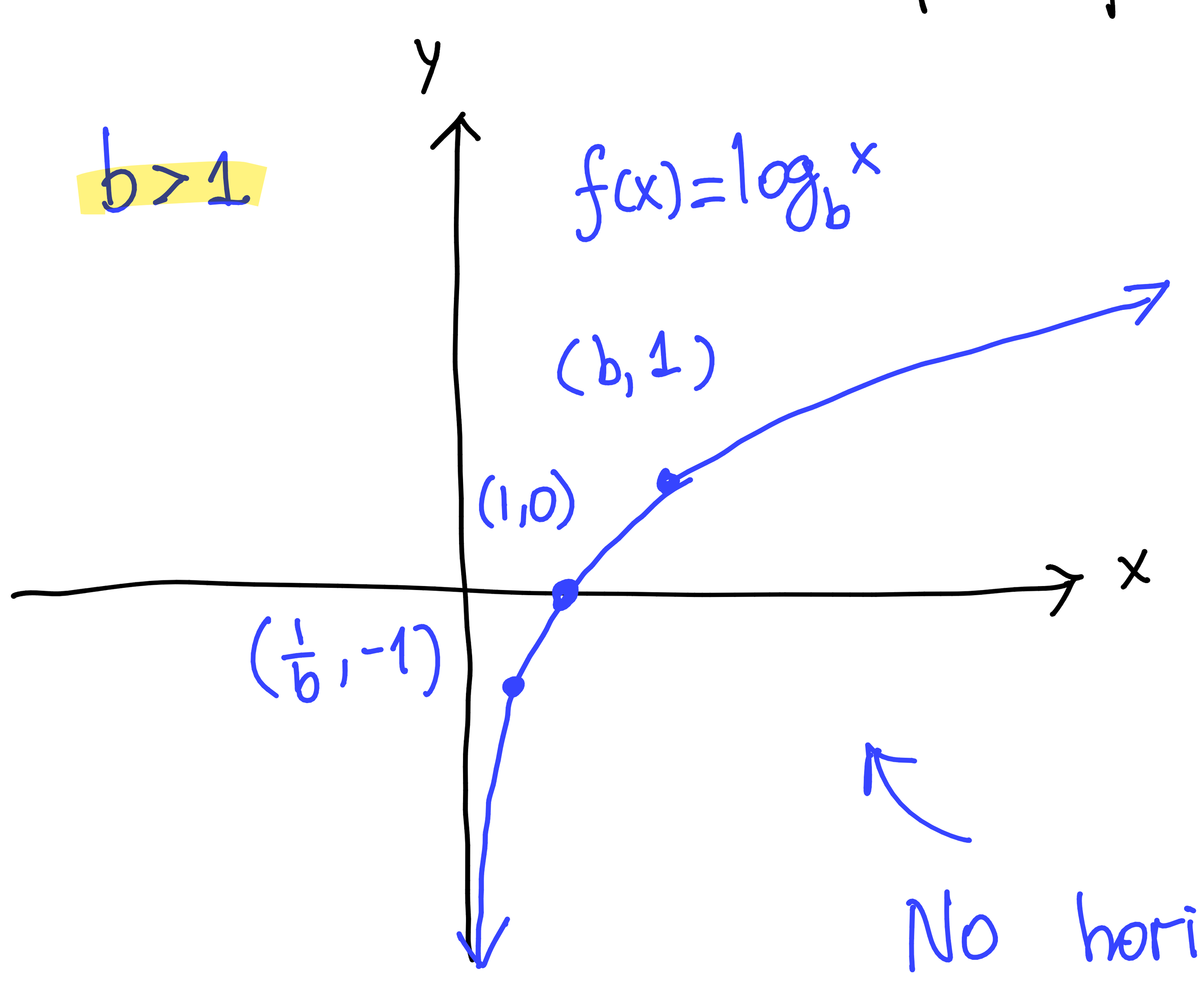
Important bases

(1) $\log_{10} x = \log_{10} x$. $\log(100) = 2$, $\log(1{,}000{,}000) = 6$, $\log\left(\frac{1}{1{,}000}\right) = -3$.

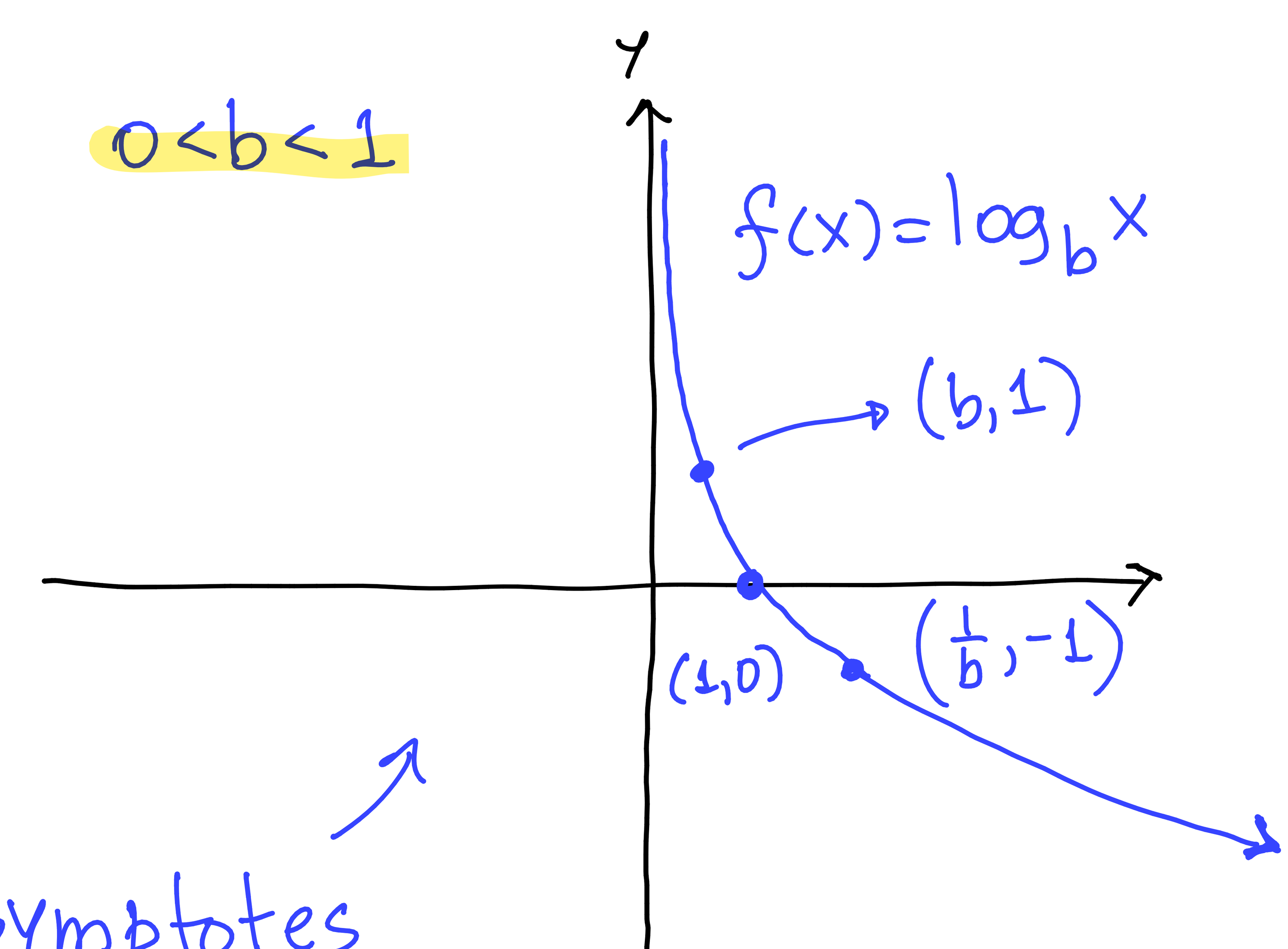
(2) $\log_e x = \ln x$. $\ln(e^6) + \ln(e^3) = \cancel{\log_e e}^6 + \cancel{\log_e e}^3 = 6 + 3 = 9$.

Graph of log function

$b > 1$



$0 < b < 1$



No horizontal assymptotes

$f(x) = \log_b(\bigstar)$ $\begin{cases} \text{Vertical assymptote } \bigstar = 0 \\ \text{Domain: } \bigstar > 0 \end{cases}$

$f(x) = \log_8(3x - 1)$

vert. assymp.:

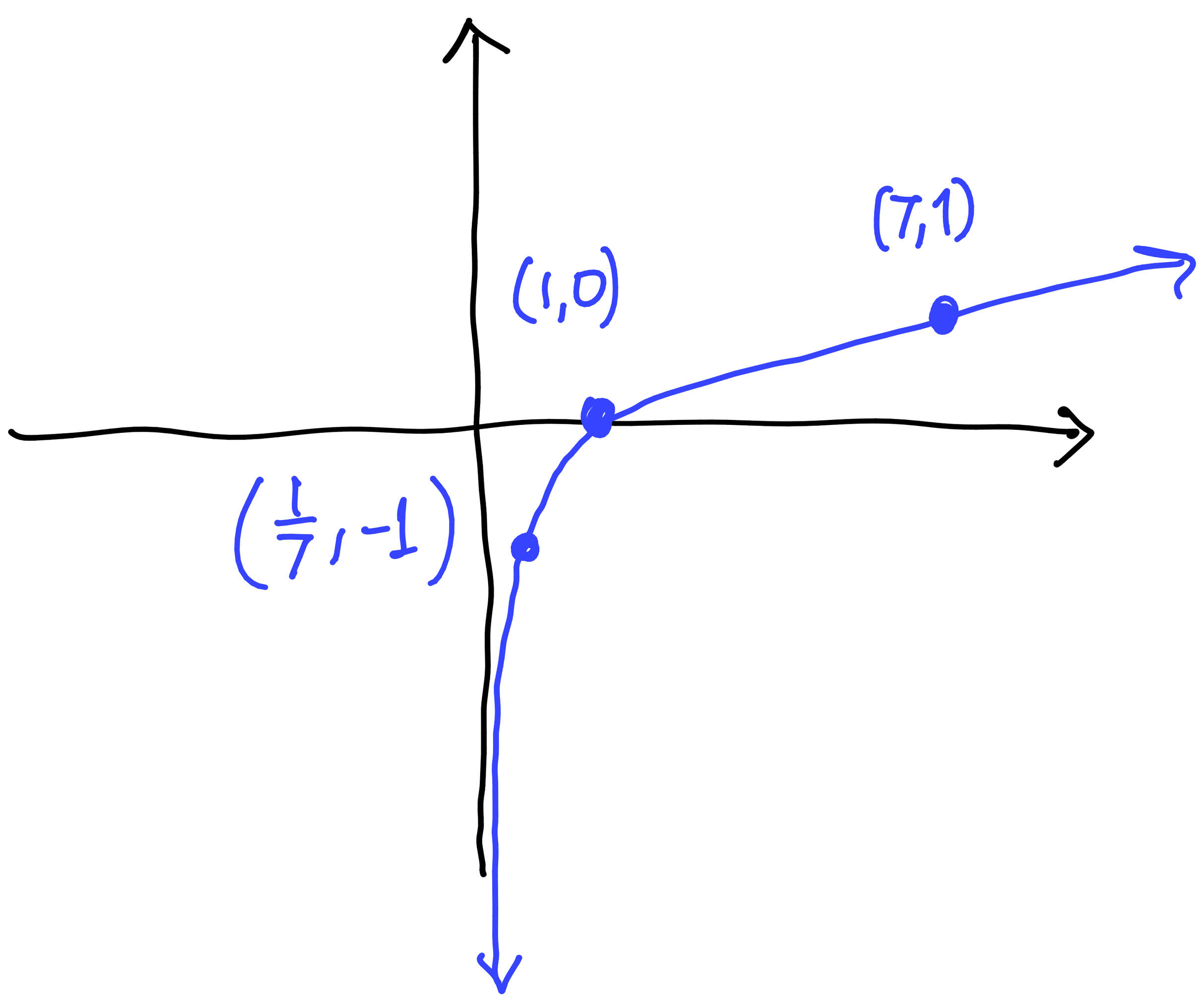
$3x - 1 = 0 \iff x = \frac{1}{3}$

Domain: $3x - 1 > 0$

$\iff x > \frac{1}{3}$

Sketching Graphs

Ex 1: $\log_7 x$.



$\log_7 1 = 0 \iff 7^0 = 1$ or $(1,0)$

$\log_7 \frac{1}{7} = -1 \iff 7^{-1} = \frac{1}{7}$ or $(\frac{1}{7}, -1)$

$\log_7 7 = 1 \iff 7^1 = 7$ or $(7, 1)$

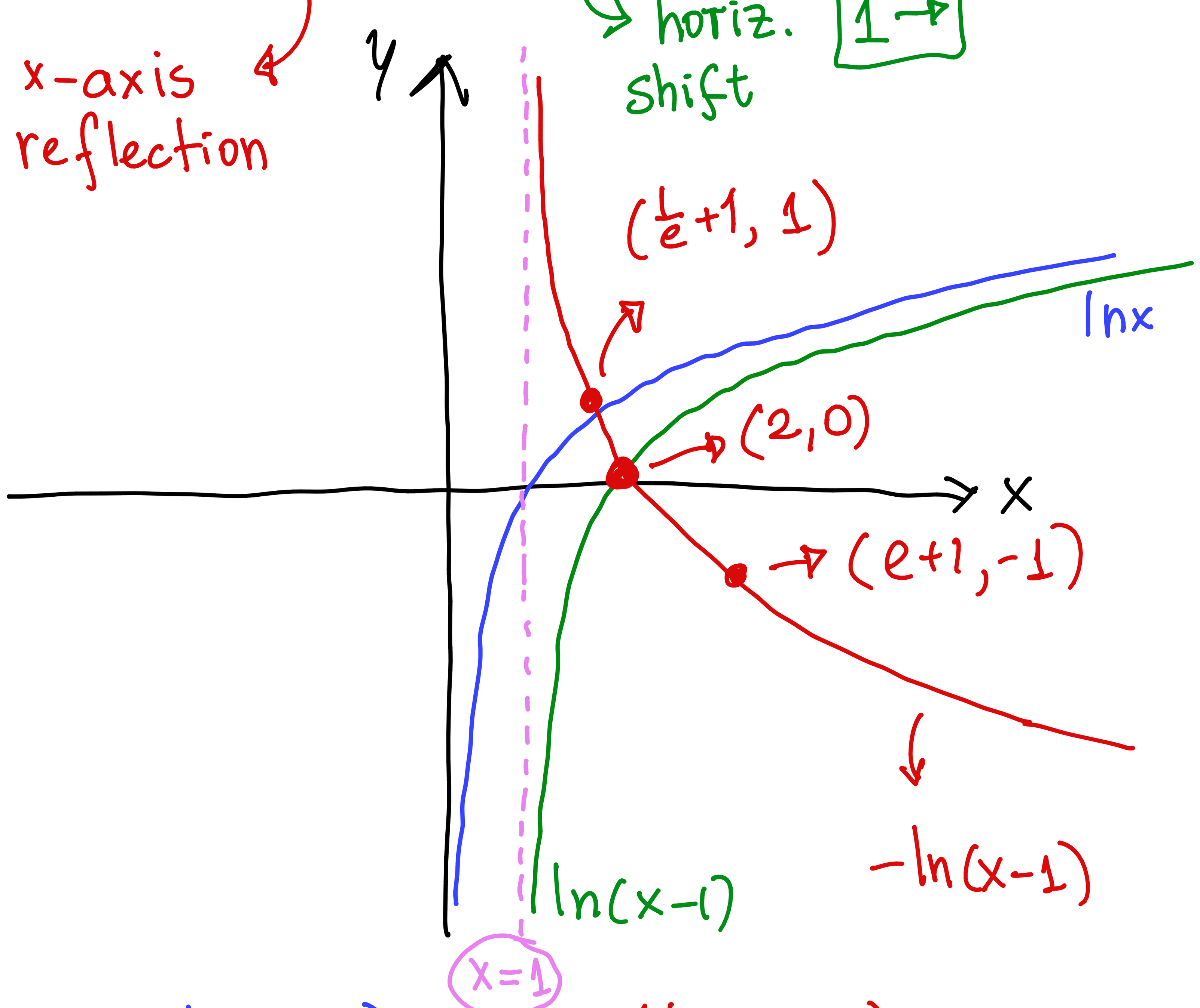
x y x y

Domain: $x > 0$. Vert. assymptote: $x = 0$

Ex 2: $-\ln(x-1)$

x-axis reflection

horiz. shift $\boxed{1 \rightarrow}$



take basic function $\ln x$

x	$y = f(x)$
$\frac{1}{e}$	-1
1	0
e	1

x: $+1$ y: $-y$

$e^{-1} = \frac{1}{e} \iff -1 = \ln \frac{1}{e}$

$e^0 = 1 \iff 0 = \ln 1$

$e^1 = e \iff 1 = \ln e$

$(\frac{1}{e}, -1) \rightarrow (\frac{1}{e}+1, 1)$

$(1, 0) \rightarrow (2, 0)$

$(e, 1) \rightarrow (e+1, -1)$

Domain: $f(x) = -\ln(x-1) \rightarrow x - 1 > 0 \iff \boxed{x > 1}$

Vert. assymp.: $\boxed{x = 1}$